

INCOHERENT EFFECT DUE TO ELECTRON CLOUD

G. Franchetti, GSI, Darmstadt, Germany; F. Zimmermann, CERN, Geneva, Switzerland

Abstract

A study of the electron-cloud pinch dynamics during a bunch passage under the effect of a single arbitrary-order multipole was presented earlier [1]. However, in a realistic situation, the proton beam will not be located in the center of the vacuum chamber. If the beam is offset a new pinch regime is encountered, where feed-down effects and asymmetry of pinch density render the dynamics more challenging. More recently we initiated an approach to investigate the dynamics of the field created by the pinched electrons, including transverse displacements the beam, the vacuum chamber of the surrounding magnet [2]. In the present paper we continue this study and develop a field map approach to the modeling of the proton beam dynamics under the effect of the pinched electrons. We compute the detuning with amplitude under various conditions through tracking simulations and compare the results with a static prediction based on the gradient of the electric field experienced on the beam axis.

INTRODUCTION

The mechanism underlying the incoherent electron-cloud effects which give rise to emittance growth or poor beam lifetime has been discussed in several papers [3, 4]. Typically a transverse detuning together with a transverse-longitudinal coupling gives rise to a periodic resonance crossing. For bunched beams, resonance crossing is of relevance when stable islands are driven into and out of the beam core. For the case of space charge the amplitude dependent detuning is created by the beam field itself and, therefore, normally shows a maximum at the beam center, which scales, for a transverse Gaussian distribution as $\Delta Q_x \sim \Delta Q_{x0}/(1 + I_x/(4\epsilon_{x0}))$, with I_x the Courant-Snyder (action) variable, ϵ_{x0} the rms beam emittance, and ΔQ_{x0} the maximum tune-shift. The detuning due to space charge decreases steeply with the inverse square of the transverse oscillation amplitude. For the case of the electron-cloud pinch, discussed in this paper, the situation is more complicated. The electron pinch produces a complex structure of localized high density peaks, that change according to the longitudinal position along the bunch. The electron cloud structure resulting from the pinch process affects the proton dynamics, also in this case, by creating an amplitude dependent detuning (coupled with the longitudinal motion), and a web of structure resonances [3]. We here present the characterization of the electron pinch responsible for the incoherent effects, and, in particular, study the effect of a beam displacement.

ELECTRON-PINCH CHARACTERIZATION

From the point of view of beam degradation, the detuning at the beam core, assumed to be located at $(\Delta x_c, \Delta y_c)$ due to a non-zero closed orbit, is of key relevance for the process of resonance crossing. The offsets Δx_c and Δy_c are the horizontal and vertical displacements of the beam with respect to the center of the vacuum chamber, respectively. In other words, although the electron-cloud structure is quite rich and complex, only the (maximum) detuning on the beam (z) axis is relevant for identifying the start of any diffusion process. In terms of proton dynamics the detuning is related to the gradient of the force generated by the structured distribution of pinched electrons: a highly localized peak of electrons at $(\Delta x_c, \Delta y_c)$ will certainly produce a higher detuning than if located at $(\Delta x_c + 4\sigma_r, \Delta y + 4\sigma_r)$.

We check this concept by investigating the detuning created by a macro-electron displaced from the center of the particle oscillations. This macro-electron can be thought of as one of the multitude composing the the electron cloud. By studying the detuning created by one macro-electron, modeled as a thin rod arbitrarily displaced transversely, we control the validity of our ansatz. We proceed by displacing a macro-electron along the x axis and computing the maximum detuning suffered by a test beam particle launched with initial conditions $0 < x/\sigma_{EC} < 10$ and $p_x = 0$, where σ_{EC} is the transverse rms size of the macro-electron rod. The macro-electron is taken to create a transverse electric field equal to that of an axi-symmetric Gaussian distribution with rms radius σ_{EC} . For each position of the rod, x_{rod} , we compute the maximum tune experienced by test beam particles with initial positions in the range $0 < x/\sigma_{EC} < 10$, as well as the tune shift of a beam test particle located near the origin. In Fig. 1 the black curve

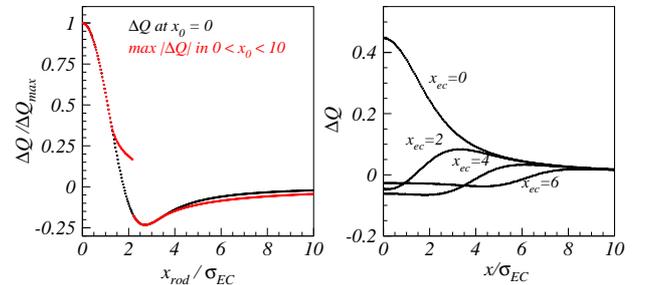


Figure 1: Left: Comparison of the detuning created on the closed orbit with the maximum detuning experienced by a test particle. Right: example of detuning of particles at different initial position for different position of the rod.

shows the tunes computed close the origin, while the red curve presents the maximum tune obtained by test particles with initial position in the range $0 < x/\sigma_{re} < 10$. We find that for nearly all rod positions the detuning on the closed orbit (here taken to be zero) is close to the maximum detuning experienced. An exception is in the range $1.5 < x/\sigma_{re} < 2$.

These considerations suggest a criterion to quantify the relevance of the electron pinch for the beam dynamics. Namely we can use as indicator of the importance of the electron pinch the gradient of the electric field created by the pinched electron distribution at the transverse position $(\Delta x_c, \Delta y_c)$ (the beam closed orbit) for several z locations along the bunch. However, even by using this criterion it is not easy to compare the effect of beam mismatch because of the presence of several peaks along z . As a first approach we only consider the maximum gradient found along z at $(\Delta x_c, \Delta y_c)$, and compare it with the initial value due the unperturbed electron distribution from before the start of the pinch.

The force of the electrons is computed assuming each macro-electron to be an infinitely long thin wire so that the force scales as $1/r$. To prevent artificial effects a cut-off is implemented. The electric field on (x, y) , $E_x(x, y)$, and $E_y(x, y)$ is computed by summing up all the forces exerted by all electrons except for those located inside a circle of radius $r_{\min} = 0.05\sigma_r$ centered at (x, y) . For N macro-electrons uniformly distributed inside a cylinder of radius R we find a gradient $dE_x/dx \propto 2N/R^2$. We cut off all particles that can create a gradient larger than this value since the gradient field created by one macro-electron is $dE_x/dx \propto 2/r^2$. The minimum radius is $r_{\min} = R/\sqrt{N}$. For $R = 10\sigma_r$, and $N = 5 \times 10^5$ we find $r_{\min} = 0.014\sigma_r$, and for safety we take $r_{\min} = 0.045\sigma_r$. The gradient is computed as $dE_x(x, y)/dx = [E_x(x + \Delta x, y) - E_x(x - \Delta x, y)]/(2\Delta)$ with $\Delta = 0.1\sigma_r$. A similar definition is used to compute $dE_y(x, y)/dy$.

Simulation condition

In the following we consider the simulations of electron pinch under the passage of an LHC proton bunch with transverse rms size σ_r of 0.88 mm, an rms bunch length σ_z of 11.4 cm, a bunch population $N_p = 1.15 \times 10^{11}$ protons, and a beam energy of 450 GeV. The bunch longitudinal distribution is Gaussian. The initial electron distribution is uniform inside a circle of radius $R = 10\sigma_r$, and it is always considered centered in the vacuum chamber, the number of macro-electrons is $N = 5 \times 10^5$. With respect to this reference frame we will displace the beam by $\Delta x_b, \Delta y_b$, or displace a magnetic element, such as a quadrupole, by $\Delta x_q, \Delta y_q$.

CHARACTERIZATION IN A QUADRUPOLE

Displacing a quadrupole

We consider here the effect of displacing a quadrupole by $\Delta x_q, \Delta y_q$, meaning that the center of the quadrupole is shifted with respect to the vacuum chamber which defines

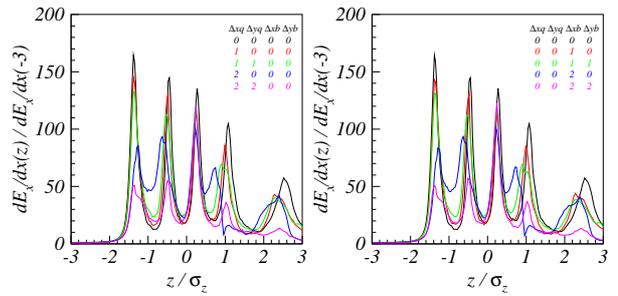


Figure 2: Left: Gradient of electron cloud induced electric field along the bunch. The colors refer to different degrees of displacement of the quadrupole as indicated by the table inserts. Right: Dependence of the pinch gradient at the beam center as a function of the beam displacement $\Delta x_b, \Delta y_b$.

the location of the initial distribution of electrons co-axial with the beam. In Fig. 2 (left) we show the gradient at $(\Delta x_c, \Delta y_c)$ along z for 4 different displacements of the quadrupole (according to the table in the picture) in units of beam σ_r .

The bunch considered is the LHC type bunch, and for the case of the quadrupole on axis each spike represents the development of a consecutive electron pinch. We see that the strength of the pinch in terms of field gradient is 100 times larger than the effect produced by the (uniformly distributed) electrons at the beginning of the bunch passage. The picture shows that when increasing the displacement of the quadrupole the first peak reduces in strength the more the quadrupole is displaced from the central position. The situation is complex: by shifting the quadrupole the field acting on the electrons acquires a dipolar feed-down component.

Displacing the beam

The shift of the beam with respect to the vacuum chamber center, is instead equivalent to the shift of the origin of the pinch: therefore, the evolving distribution of the electrons is now shifted off axis following the transverse center of the beam. The pinch process should in this case be affected by the asymmetry of the initial electron distribution, with respect to the displaced center of the beam. However, the conflicting effects of the quadrupole forces centered at the origin of the beam pipe and the Coulomb attraction towards the shifted bunch also play a crucial role. We expect that a shift of the beam axis will significantly reduce the effect of detuning experienced as a function of position along the bunch. Figure 2 (right) presents this effect when displacing the beam by the same amounts as considered for the quadrupoles in the left figure. The comparison shows that there is no significant difference in the electron-cloud gradient experienced on the beam axis when displacing either the beam or the quadrupole by the same amount. A similar finding is obtained for the case of dipole magnets.

COMPARING THE EFFECT OF SHIFT IN DIFFERENT ELEMENTS

We here discuss the effect of the beam displacement in several basic accelerator elements such as 1) drift, 2) dipole, and 3) quadrupole. The study is made by plotting the maximum gradient along z for several beam displacements. In Figs. 3 we show the result of the simulation study where $\text{Grad} \equiv \sqrt{M_x^2 + M_y^2}$, and $M_x = \max\{\frac{dE_x(x,y,z)/dx}{dE_x(x,y,-3\sigma_z)/dx} : -3\sigma_z < z < 3\sigma_z\}$, and $M_y = \max\{\frac{dE_y(x,y,z)/dy}{dE_y(x,y,-3\sigma_z)/dy} : -3\sigma_z < z < 3\sigma_z\}$.

Discussion:

Drift. In Fig. 3 (left) we explore the maximum pinch for a beam a displacement in the range $-4\sigma_r, 4\sigma_r$ for x and y planes. We notice that for the pinch in a drift the maximum gradient is located in a circular region of radius $2\sigma_r$ where the relative gradient is ~ 430 times larger than the initial gradient created by the uniformly distributed electrons. The circular region arises due to the shape of the initial electron distribution, a circular uniform distribution of radius $R = 10\sigma_r$. We conclude that a displacement within a radius of $2\sigma_r$ does not affect the electron pinch in the drifts. (In case we had a larger initial electron distribution this radius would be larger).

Dipole. Simulation study presented in [2] shows that the normalized gradient is around ~ 25 in a dipole and that this value is constant in the full region of the pipe ($10\sigma_r$). This result is a consequence of the physical process creating the pinch in a dipole. As discussed in Ref. [4] the electrons are constrained to vertical motion by the strong dipolar magnetic field. Hence the electrons participating in the pinch on the longitudinal axis are mainly those in the neighborhood of a vertical slice of electrons (centered around $x = 0$) passing through the longitudinal axis. The number of these electrons is significantly smaller than those participating in the pinch for the drift case. This explains the relative weakness of the normalized gradient, which now is 25 compared with 430 for the case of the drift.

Quadrupole. The pinch in a quadrupole shows maximum gradients that are weaker with respect to those of the drift case; see Fig. 3 (right). The picture exhibits the symmetry imposed by the quadrupole, but the maximum gradient is now smaller because the effect of the quadrupole is absent only for electrons located on the diagonals [1]. For all other electrons the force of attraction towards the center of the bunch is affected by the force of the quadrupole that pushes the electrons away, thereby diminishing the pinch effect and hence the gradient on the beam axis.

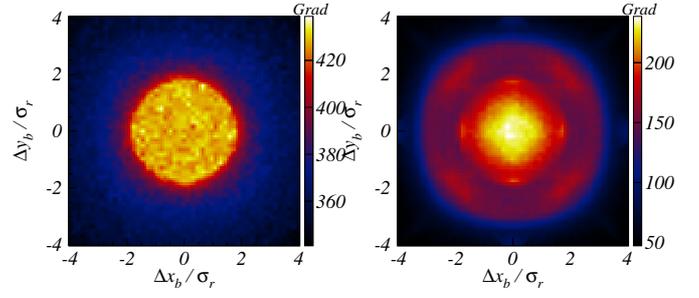


Figure 3: Maximum gradient for drift (left) and quadrupole (right) as function of the beam displacement ($\Delta x_b, \Delta y_b$). The gradient is normalized to the gradient created by the electrons at the beginning of the bunch passage.

For the sake of comparison and to show the complexity of the electron dynamics in Fig. 4 we present a comparison between the pinch in a quadrupole obtained for a beam on axis and for a beam shifted by $\Delta x_b = 3\sigma_r$.

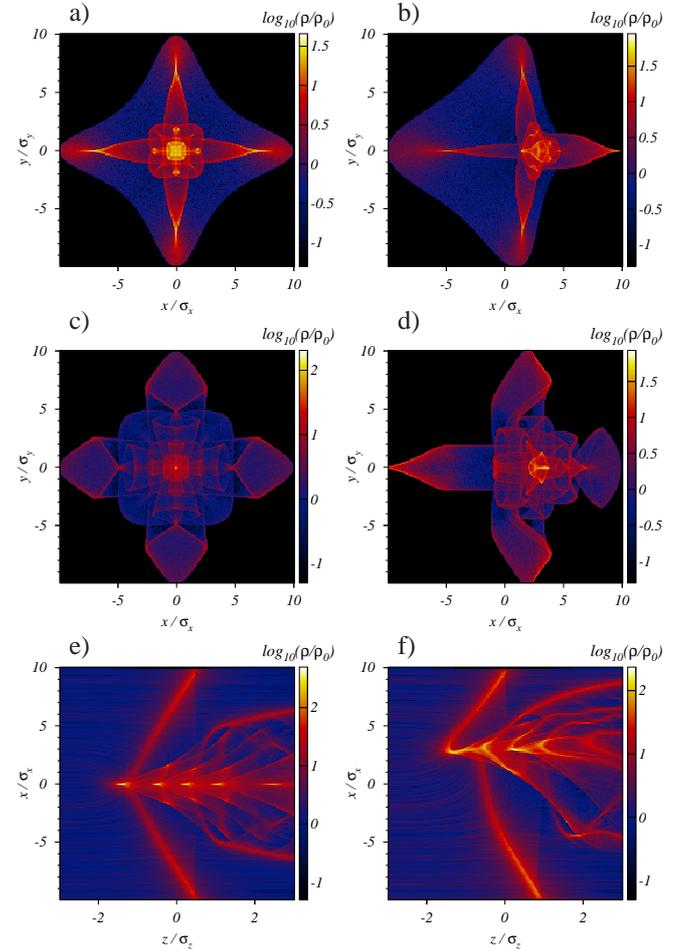


Figure 4: Electron density enhancement for the proton beam on axes (left column), and the same simulation with the beam shifted of $\Delta x_b = 3\sigma_r$ (right column). ab) the $x - y$ plane at $z = 0$; cd) the $x - y$ plane at $z = 1$; ef) the $z - x$ plane at $y = 0$.

Note that all this discussion is based on static indicators, which rely on the computation of gradients. Therefore the next step is computing the tune shift from the particle dynamics.

COMPARISON WITH THE DYNAMICS

The implementation of a realistic modeling of the EC incoherent effects requires that at each location where the electron cloud is found the pinch dynamics should be computed including the local properties of the lattice as well as the electron cloud properties. The amount of CPU power necessary for this task is extraordinary, and in order to simplify the study, here we consider an approximate approach, which nevertheless distinguishes the pinch dynamics for drifts, dipoles, and quadrupoles.

The procedure we use consists in the following steps:

1) We compute the "normalized transverse force" E_x, E_y created by electrons at each location of the bunch. By normalized it is meant that the electric field E_x, E_y is computed for a reference charge density assigned to each macro electron. As consequence the realistic force is obtained by re-scaling this force of a proper factor F , proportional to the electron cloud density.

2) The field E_x, E_y is stored as function of x, y, z on a $200 \times 200 \times 200$ grid that include the bunch itself. The grid extends to $[-10\sigma_r, 10\sigma_r]$ in both transverse axis, while on the longitudinal axis it extends in the range $[-3\sigma_z, 3\sigma_z]$.

3) The actual force on a proton when it passes through the electron cloud at the longitudinal position z (in the bunch reference frame), is obtained via tri-linear interpolation from the grid data.

4) The previous procedure is applied for a pinch of electrons in a drift, dipole, and a quadrupole. We define in this way 3 new elements "EC kick" which are consistently applied in the neighbor of each element of a circular accelerator structure.

This procedure at the moment remains incomplete as it does not take into account of the differences in optics at different locations where electron pinch will take place. Clearly these optics differences are responsible of deforming the transverse section of the proton beam which consequently will produce a "deformed" electron pinch. Hence it becomes necessary, but it is left to future studies, to establish if there is a scaling property of the structure of electron cloud with β_x, β_y at the location in which the pinch takes place.

Detuning before the pinch

The localized electrons are initially, at $z = -3\sigma_z$ in the bunch reference frame before the pinch evolves, uniformly distributed. The effect of the electrons on a proton, neglecting other fields, is given by

$$x'' = \frac{e}{m_p \gamma c^2} E_{x,ec},$$

but for electrons uniformly distributed in a circular pipe the electric field is given by $E_{x,ec} = x\rho/(2\epsilon_0)$. Therefore the

effect on the dynamics inside the electron cloud region is

$$x'' = \frac{e}{m_p \gamma c^2} \frac{\rho}{2\epsilon_0} x.$$

We find therefore that an electron cloud of density \tilde{n}_e and charge density $\rho = e\tilde{n}_e$ localized in a region of length L produces on the dynamics an integrated effect equivalent to a quadrupole magnet K_{ec} of strength

$$K_{ec} = \frac{e}{m_p \gamma c^2} \frac{\rho L}{2\epsilon_0}$$

The detuning in the x -plane becomes

$$\Delta Q_x = \frac{\beta_x}{4\pi} \frac{e}{m_p \gamma c^2} \frac{\rho L}{2\epsilon_0}.$$

For LHC at $\gamma = 450$ we find $\Delta Q_x = 1.71 \times 10^{-21} \beta_x \tilde{n}_e L$.

Due to the method for implementing the electron cloud Coulomb force we find that the electric field scales linearly as $F10^4(x/\sigma_r)$. This form stems from how the data are computed: we fill a circle of $10\sigma_r$ radius with 5×10^4 macro-electrons and the electric field is stored in a grid large $10\sigma_r$. The number 10^4 denotes the gradient in normalized units, and F is a factor used for tuning the charge of a macro-electron to a given electron cloud density, with a specified detuning. Considering that $\sigma_r = \sqrt{\beta_x \epsilon_x}$ we find that the gradient of the force is $K_x = F10^7/\sqrt{\beta_x \epsilon_x}$ where the emittance is in units of mm-mrad, and beta in meters. Consequently the detuning created by one electron cloud kick is $\Delta Q_x = \beta_x K_x/(4\pi) = (F\sqrt{\beta_x}10^7)/(\sqrt{\epsilon_x}4\pi)$. By using this formula we have computed the theoretical detuning expected from 3 different EC kicks (dipole, quadrupole, drift) each applied to different positions of the lattice.

In order to verify the correct use of the electron generated electric field maps, we compute the detuning induced by the EC by freezing the longitudinal motion and taking a test particle located at $z = -3\sigma_z$ in the proton bunch. We consider here the case of an artificially enhanced electron cloud density to the purpose of better computing the particle detuning. The parameter F is taken to $F = -10^{-10}$ for all 3 types of EC kick to which correspond an artificially large electron cloud density ($\tilde{n}_e \sim 10^{16} \text{ m}^{-3}$). The detuning over the full bunch length is shown in Fig. 5 for LHC lattices with only one EC kick respectively in a Dipole (B), Drift (D), and Quadrupole (Q).

Characteristic tune shifts are also summarized in Tab. 1: ΔQ_T is the theoretical tune shift and ΔQ_S the one from simulations, both computed at $z = -3\sigma_z$. In the table we also report the maximum tune shift ΔQ_{\max} along the z axis. The ratio $\Delta Q_{\max}/\Delta Q_S$ should be equal to $\text{Grad}/\sqrt{2}$ as previously defined (also reported in Tab. 1). For nearly all cases we find that $\Delta Q_{\max}/\Delta Q_S$ is a factor ~ 1.5 smaller than the correspondent column with "Grad". At the moment we cannot explain the origin of the factor 1.5.

EXAMPLE FOR LHC

As an example we implement EC kicks in all quadrupoles and bends of LHC, here with $F = -2 \times 10^{-15}$,

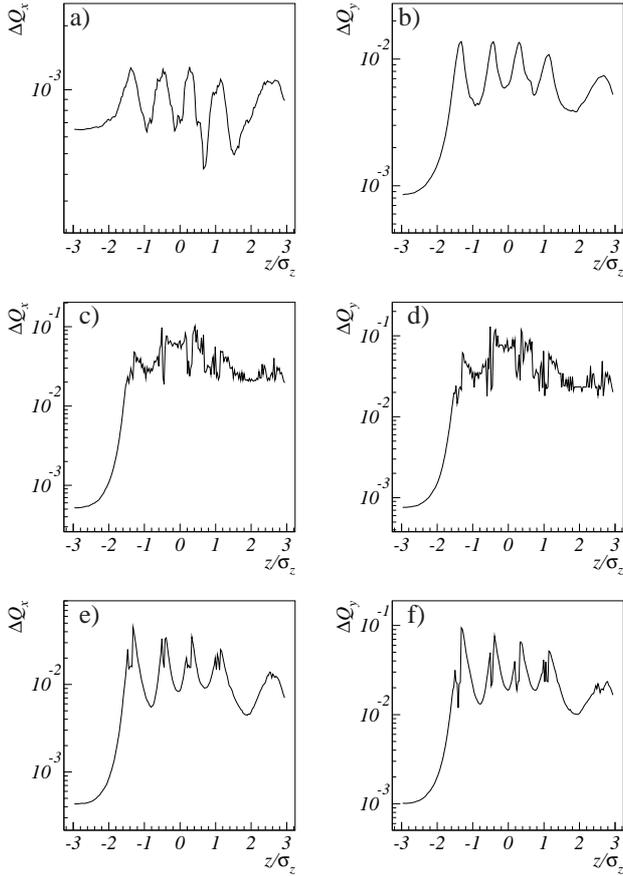


Figure 5: Detuning along the bunch for test particles at transverse amplitude of $0.1\sigma_r$. The right column is the horizontal tune, the left column is the vertical tune. ab) one EC kick in a dipole; cd) one EC kick in a drift; ef) one EC kick in a quadrupole.

Table 1: Summary of the detuning for a single EC kick. Q_T is the theoretical detuning, Q_S is the detuning obtained by simulations.

		β m	ΔQ_T $\times 10^{-4}$	ΔQ_S $\times 10^{-4}$	ΔQ_{max}	$\frac{\Delta Q_{max}}{\Delta Q_S}$	Grad
B	x	55.7	5.9	6.5	0.0012	1.94	27
	y	110.6	8.3	8.5	0.013	16.1	27
D	x	59	6.11	5.2	0.101	195	420
	y	105.6	8.17	7.6	0.129	170	420
Q	x	33.6	4.6	4.32	0.045	104	230
	y	162	10.1	10.1	0.094	93	230

therefore assuming the initial electron cloud density consistent with $\tilde{n}_e \sim 2 \times 10^{10} \text{ m}^{-3}$. The peak detuning is expected as $\Delta Q_{x,peak} = 0.0046, \Delta Q_{y,peak} = 0.0045$. A bunch made by 10^4 macro particles is tracked for 2×10^6 turns equivalent to 3 minutes beam run in LHC. The beam evolution is shown in Fig. 6. Understanding the origin of the coupling between the two transverse emittances is left

for future studies.

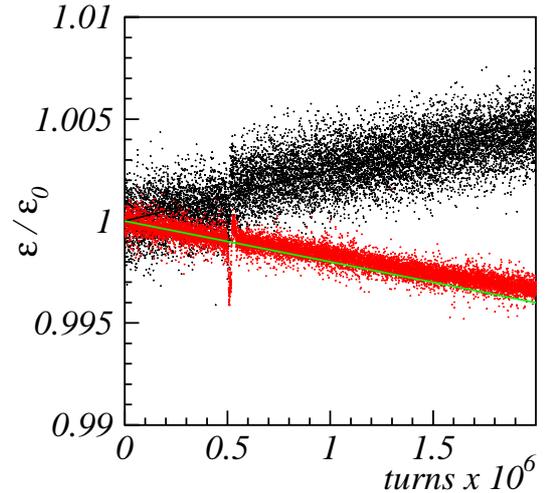


Figure 6: Example of emittance evolution in LHC for an electron cloud applied to each quadrupoles, bends, and the short straight section.

CONCLUDING REMARKS

The first part of this study shows that from r displacements of the beam with respect to the center of the vacuum chamber no significant effect on the tune shift is expected as long as the displacement stays within a radius of $1\sigma_r$. For larger values the most significant reduction is found in the quadrupoles. The results found also allow comparing the relative importance of localized electrons along the machine: They show that the gradient in a quadrupole is significantly larger than in a dipole.

In the second part we implemented a field map approach to the description of the incoherent field generated by the pinched electrons. We benchmarked the tune shift induced by the electrons on the protons prior to the pinch, for which analytic descriptions and simulations are in good agreement. In further investigations the maximum tune shift normalized to the initial tune shift seems to be a factor ~ 1.5 smaller than the value expected from a static indicator. The discussion on the origin of this factor and the discrepancy of one case requires a systematic investigation of the peak detuning from the field map algorithm, including grid size, interpolation strategy, cut off criteria, and noise or errors introduced by it.

This work was supported, in parts, by the European Commission under the FP7 Research Infrastructures project EuCARD, grant agreement no. 227579.

REFERENCES

- [1] G. Franchetti and F. Zimmermann, Proc. IPAC2011, San Sebastian, Spain. MOPS001. p. 586.
- [2] G. Franchetti and F. Zimmermann, Proc. IPAC2012, New Orleans, USA. WEPPR047.
- [3] E. Benedetto *et al.*, PRL 97:034801 (2006).
- [4] G. Franchetti *et al.*, PRSTAB 12, 124401 (2009).